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# Families as Neighbors in Extra Dimension

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## Abstract

We propose a new mechanism for explanation of the fermion hierarchy without introducing any family symmetries. Instead, we postulate that different generations live on different branes embedded in a relatively large extra dimension, where gauge fields can propagate. The electroweak symmetry is broken on a separate brane, which is a source of exponentially decaying Higgs profile in the bulk. The resulting fermion masses and mixings are determined by an exponentially suppressed overlap of the fermion and Higgs wave functions and are automatically hierarchical even if all copies are identical and there is no hierarchy of distances. In this framework the well known pattern of the “nearest neighbor mixing” is predicted due to the fact that the families are literally neighbors in the extra space. This picture may also provide a new way of a hierarchically weak supersymmetry breaking, provided that the combination of three family branes is a non-BPS configuration, although each of them, individually taken, is. This results in exponentially weak supersymmetry breaking. We also discuss the issue of embedding identical branes in the compact spaces and localization of the fermionic zero modes.

# 1 Introduction

Large extra dimensions may help in understanding the hierarchy between the Planck and weak scales [1]. In the present paper we will concentrate on the hierarchy of the fermion masses which is another mystery in the standard model (SM). One possible approach relies on (spontaneously broken) flavor symmetries. But this does not really answer the question, rather brings it at a different level. Instead of explaining the hierarchy of the Yukawa couplings, now one has to explain the hierarchy of the breaking scales.

In the present paper we will adopt a different attitude. We assume that three SM families are identical, the difference in their masses is simply because they happen to live in different places in the extra space. More precisely, we assume that the original higher dimensional theory admits, as its solution, a brane with localized fermions with quantum numbers of one SM generation. Multiple brane states will then generate  $\nu$  identical copies of fermions,  $\nu$  generations of the standard model. Due to obvious reasons we will take  $\nu = 3$  in our discussion. It is clear that if the quarks and leptons are to come from different branes, then the gauge fields must freely propagate in the interbrane space. The transverse volume covered by gauge fields may be a world-volume of a “fatter” brane, or simply a compactified dimension. In any case there is an upper bound on a linear scale associated with this volume,  $L \sim 1/(1\text{TeV})$ .

The crucial question in this picture is where does the electroweak symmetry breaking happens? We postulate that the vacuum expectation value (VEV) of the Higgs field is induced due to the presence of a separate brane. The latter acts as a source for the Higgs VEV in the perpendicular direction, so that the Higgs VEV decays exponentially away from the source,

$$H \sim e^{-r/r_0}, \quad (1)$$

where  $r$  is the distance from the source brane.

Thus, there is a nonzero Higgs profile in the bulk, and this will generate masses of the SM fermions localized on other branes. In this way the mass of the SM fermions will be determined by the overlap of its wave function (squared) with the Higgs profile. This can be of order one for the nearest brane, but exponentially suppressed for more distant neighbors.

Note that there is no need to postulate a hierarchy of distances between immediate neighbors. In any case, the hierarchy of the fermion masses is guaranteed. Note also that there is an inevitable correlation between the masses and mixings. The mixing between the fermions is suppressed by the overlap of two (distinct) wave functions with the Higgs profile. As a result the nearest neighbors will mix stronger than the next-to-nearest, and so on. This pattern is well-known experimentally.

Other input assumptions are more or less standard for the approach with the large (compact) extra dimensions. Yet it is worth discussing them in brief.

We will consider one extra dimension, so that the space has the topology of  $M_4 \times S$ . The size of the extra dimension  $L$  is assumed to be much larger than  $M_{\text{Pl}}^{-1}$  and the brane width  $\delta$ . Gravity is weak at these distances, and plays essentially no role provided there are other (passive) extra dimensions in which our construction is embedded.<sup>1</sup> A microscopic Planckian theory descends to distances  $L$  in the form of some field theory and the given geometry of space-time. This field theory is responsible for the build-up of the branes, with the zero modes as discussed above. The field(s) that “build” the walls are distinct from the matter and Higgs fields, they have to be introduced for the wall-building purpose.

Presumably, the characteristic size of our extra dimension, should be somewhat smaller than inverse TeV, due to reasons associated with the flavor violation. In the theories with a low fundamental scale ( $M_{\text{Pf}}$ ), there is a potential danger of higher-dimensional operators that lead to a flavor violation in the low-energy processes through the higher dimensional operators suppressed by powers of  $M_{\text{Pf}}$ . These can be, in principle, controlled by gauging *non-Abelian* flavor symmetries in the bulk [2]. Flavor-violating exchange by the bulk flavor gauge fields or by the scalar flavons can be adequately suppressed [2]. In our framework, however, there can be an additional source of flavor violation due to the exchange of the ordinary gauge fields [3].<sup>2</sup> This exchange is suppressed by the size of extra dimension versus the localization width of the fermions, and may require the size of extra dimension to be below  $1/(1000\text{TeV})$ . In our discussion, we will assume this bound to be satisfied, and keep the size as a free parameter. Of course, making  $L$  small implies increasing the cutoff of the theory, and at some point it will reintroduce the hierarchy problem. Therefore, the issue of the low-energy supersymmetry (broken at the scale  $\ll M_{\text{Pf}}$ ) may become important in our framework.

In this respect it is interesting that localization of families on the identical branes may automatically lead to a novel mechanism of the *exponentially weak* supersymmetry breaking, provided that each individual family brane is a BPS state, whereas their combination is not. (Supersymmetry breaking on non-BPS branes was suggested in [5].) In Sec. 6 we will formulate a general sufficient condition for such a breaking.

Before proceeding to a more detailed consideration let us summarize crucial differences of our scenario from the existing alternative high-dimensional mechanisms of the fermion mass generation. In [6] the hierarchy of the fermion masses was generated by invoking global flavor symmetries broken on a set of distant branes. Each of the distant branes was responsible for the breaking of a particular subgroup of the full flavor group. This breaking then was communicated (“shined”) to the standard model fermions via a set of bulk messenger fields, in some representation

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<sup>1</sup>The graviphoton is eliminated by the overall zero mode associated with the breaking of the translational invariance in the fifth direction.

<sup>2</sup>This is in contrast to other (unbroken) global symmetries of the standard model (such as the baryon number), which, in principle, can be protected by separating quarks from leptons in the extra space [4].

of the flavor group. Although the SM fields were localized on the same brane, the resulting pattern of flavor symmetry breaking can still be hierarchical if the branes responsible for different breakings are located at different distances.

Note that in this picture it is essential to have a flavor symmetry, as well as a set of the flavor-breaking branes with a variety of sets of the VEV's, plus a sector of the bulk messenger fields charged under the flavor group. In our scenario there is no need to postulate any flavor symmetry at all. No messenger fields are needed. The only Higgs field that acquires an expectation value is the standard model electroweak Higgs. It is impossible to avoid the hierarchical pattern of fermion masses, except for the unlikely case when all the three branes are stabilized right on top of each other. As will be discussed below, such stabilization is very difficult to achieve in practice, unless an unnatural distinction among the family branes is introduced.

## 2 Fermion Hierarchies from extra dimensions

In this section we will discuss some model-independent features of the fermion masses in our framework and show why a hierarchical pattern is inevitable. Although this will not be crucial for our purposes, for simplicity and economy, we assume that all the standard model fermions are generated from a single progenitor family in the original five-dimensional theory. The corresponding Yukawa couplings in the five-dimensional action are

$$S = \int d^5x g_f H \bar{f} f_c \quad (2)$$

where  $H$  is the five-dimensional Higgs field and  $f$  and  $f_c$  are the five-dimensional fermions which give rise to the four-dimensional chiral  $SU(2) \otimes U(1)$ -doublet and singlet fermions, respectively.

Now, the fact that the SM fermions are localized zero modes on the branes, means that the five-dimensional fermionic fields allow for the expansion of the form

$$f = \sum_i \Omega_i(y - y_i) f_i(x_\mu) + \dots \quad (3)$$

where  $y$  is the fifth coordinate and  $\Omega_i(y - y_i)$  are localized functions at  $y_i$ , with an exponentially decaying profile. (It is assumed that  $|y_i - y_j| \ll L$ . At  $|y - y_i| \sim (L/2)$  the exponential decay regime changes, see below.)

The functions  $f_i$  are zero modes of the four-dimensional Dirac operator. In this way the expansion (3) describes the zero-mode fermions of three standard model generations localized at the hyperspaces  $y = y_i$  in the bulk.

However, because of the finite distance between the branes these states are not completely orthogonal – there is a nonzero overlap between the wave functions. This amounts to a nonzero but small mixing between the fermions. What is interesting, there is a nontrivial correlation between the fermion mixing and their masses. This is clear from the way they are generated. The source of the masses is the expectation value of the Higgs field, which we assume is induced on a separated “source” brane,

located at some point  $y = 0$ . The Higgs VEV is maximal at the brane and decays exponentially in the bulk. Outside the Higgs VEV-generating brane, i.e. at  $|y| > \delta$

$$H(y) = v \left( 2e^{L/2} \right) \cosh \left[ a \left( y - \frac{L}{2} \right) \right] \quad (4)$$

where  $a$  is the mass scale defining the inverse brane “thickness” and  $v$  is the Higgs VEV on the brane. Assuming that all three family branes sit in the domain of the exponential decay of  $H(y)$ , the fermion masses will be determined by the overlap of the fermion wave functions with the exponential Higgs tail in the extra dimension, and, thus, by the distance from the source brane. Consider a situation when the family branes are located on the same side from the source brane (i.e. the Higgs VEV-generating brane). Then, even if they are placed in equal intervals, the hierarchical pattern of masses and mixings is guaranteed.

For illustrative purposes we can approximate fermionic wave-functions by exponential profiles

$$\Omega_i = \exp(-|y - y_i|b) , \quad (5)$$

where as in the case of the Higgs field,  $b$  is a mass scale that sets the “thickness” of the fermionic profile(s), and we assume that all fermionic branes are identical (we neglect a small distortion of the wave functions because of the non-zero overlap). Then, the fermion masses are given by the following overlap integrals:

$$m_{ij} = \int dy v \exp \left\{ -b \left( |y| \frac{a}{b} + |y - y_i| + |y - y_j| \right) \right\} . \quad (6)$$

Note that, equivalently, we could have obtained the same result by going to the effective low-energy picture. The existence of the Higgs profile (4) means that there is a four-dimensional Higgs state localized on the brane. This mode corresponds to vibrations of the condensate and, therefore, is localized on the source brane. Thus, integrating out the extra dimension we will be left with a SM-like pattern, with the hierarchically suppressed Yukawa interactions.

### 3 Higgs profiles

In this and the next sections we will consider some technical details such as the generation of the Higgs condensate on a brane as well as the embedding of the multiple fermionic branes in the compact dimension. The generation of the Higgs condensates on the brane is a frequent phenomenon, whenever there is a nontrivial coupling of the bulk scalars with the brane. For instance, we will consider a simple example of the domain wall studied in [7]. In this example there is a domain wall created by a real scalar field  $\chi$ , and another field  $H$  charged under the gauge group  $G$ . In our case this will be assumed to be electroweak  $SU(2) \otimes U(1)$ . It is essential that the Lagrangian contains interaction among these fields. In the simplest form

the (scalar sector) of the five-dimensional action can be taken as

$$S = \int d^5x \left[ |D_\mu H|^2 + |\partial_\mu \chi|^2 - \left( a(\chi^2 - \mu^2)^2 + (b\chi^2 - m^2)|H|^2 + |H|^4 \right) \right]. \quad (7)$$

For  $b\mu^2 > m^2$  the system has two ground states, with  $\chi = \pm\mu$  (and  $H = 0$  for both). Due to simple topological arguments there is a wall (brane) interpolating between the two. For  $H = 0$  the wall profile has a simple form

$$\chi = \mu \tanh(y\mu\sqrt{4a}). \quad (8)$$

Since  $\chi$  goes through zero in the middle of the wall,  $H$  can become unstable and condense on the brane. This can be simply seen by examining small perturbations  $H = He^{i\omega t}$  in the wall background. The linearized Schrödinger equation takes the form

$$\partial_y^2 H - \left( b\mu^2 \tanh^2(y\mu\sqrt{4a}) - m^2 \right) H = \omega^2 H \quad (9)$$

which clearly has an unstable eigenmode (with imaginary  $\omega$ ) for some range of parameters.

Thus, the SM symmetry is spontaneously broken on the brane but is restored in the bulk in the infinite volume limit. For the finite extra dimension, the gauge fields will get nonzero masses by interacting with the brane condensate, and the gauge symmetry will be spontaneously broken in the effective low-energy theory.

Finally, let us remark on a technical point regarding the embedding of a brane (or several branes, as opposed to the antibrane) in the compact extra dimension. This issue was discussed in great detail in [8]. The wall one may have on the cylinder is slightly different from the standard kink in the non-compact space, which interpolates between distinct vacua of the theory. To have walls on the cylinder one must assume that the fields of which the walls are built are defined on manifolds with noncontractible cycles. For instance assume that  $\chi$  is an angular variable (an “axionic” type field), defined modulo  $2\pi$ . The appropriate interaction potential is

$$V = 2M^4 \cos \chi + (b \sin \chi - m^2)|H|^2 + |H|^4 \quad (10)$$

The resulting brane is a sine-Gordon soliton

$$\chi = 4 \tan^{-1} \exp(y m), \quad (11)$$

and can be embedded in the compact space. Since  $\chi$  changes by  $2\pi$  through the soliton,  $\sin \chi$  becomes zero and destabilizes  $H$ , much in the same way as for the kink.

Moreover, as it was shown in [8], the isolated wall on the cylinder may be BPS saturated, i.e. leading to supersymmetric low-energy theory of the zero modes. But we have several branes: three “fermion” and one Higgs VEV-generating. It is natural to assume that, being considered in isolation, each wall is BPS saturated. Taken all together they need not necessarily be BPS. Hence, one can get a supersymmetry

breaking exponentially small in the parameter  $|y_i - y_j|/\delta$  where  $\delta$  is the wall “thickness.” In this way, one gets an exponential suppression of the SUSY breaking scale without any input hierarchy.

To reiterate, had we just one generation, supersymmetry will be unbroken. It is the intergenerational (interbrane) interference that makes the wall configuration non-BPS, and breaks SUSY.

This effect is independent of the other arguments presented above regarding the fermion mass hierarchy. The original masslessness of the fermions is not due to SUSY, but, rather, due to the topological property of the brane (or due to a mechanism to be discussed in Sec. 4). Note that the matter fields need not be chiral at distances  $\ll L$ , when our intermediate field theory flows to a fundamental one. The chirality of the trapped zero modes occurs as a result of the winding of the solution under consideration in the extra dimension. In this picture it is natural that the matter fermions are lighter than the sfermions.

## 4 Multiple branes in compact spaces

The fermionic fields must be localized on a number of identical stable branes, admitting the fermionic zero modes. Stability of such branes, in general, is due to some charge  $Q$ . This may be either a topological charge (e.g. in the case of the kink or soliton) or a charge with respect to some higher forms (e.g. as in the case of  $D$  branes). The corresponding flux then guarantees the stability of the brane. The same flux conservation then often forbids the embedding of the branes in the compact space, since the flux can end nowhere. One is then forced to introduce antibranes on which the flux lines may end, to balance the total charge. In our scenario we would like to avoid antibranes.

One possible way out then is to consider topological charges compatible with the compact boundary conditions, as above (see [8]). For instance, we can put arbitrary number of solitonic branes of the form (11) in the compact space.

Here we will consider an alternative way for dealing with this issue in the cases when the flux conservation is incompatible with periodicity of the space. Let such charge be  $Q$ . Then, instead of introducing an antibrane with the charge  $-Q$ , we can assume that the charge in question is Higgsed. Then the flux lines will be absorbed by the “medium,” much in the same way as the conductor absorbs the electric flux.

Let us consider the issue of the stability of such a system. As a prototype toy model consider a brane which is a source of a massless scalar field  $\phi$ . The corresponding coupling of  $\phi$  to the world-volume of the brane is

$$Q \int dx_\alpha \wedge dx_\beta \wedge dx_\gamma \wedge dx_\delta \epsilon_{\alpha\beta\gamma\delta} \phi. \quad (12)$$

In this way the brane “shines” a massless scalar field. If there are no lighter states in the theory charged under  $Q$ , the brane will be stable due to the flux conservation.

The massless field  $\phi$  satisfies the classical equation with the delta-function source in the fifth coordinate  $y$  (transverse to the brane)

$$\partial_y^2 \phi = Q\delta(y). \quad (13)$$

The solution of the above equation is evident,

$$\phi = Q|y|. \quad (14)$$

This solution shows that it is impossible to put such field on the cylinder.

Imagine now that we give a small mass  $m$  to the scalar field. This will guarantee that the flux is screened at large distances  $y \gg m^{-1}$  as

$$\phi \sim Q\exp(-m|y|). \quad (15)$$

The exponential solution above is valid for the noncompact fifth dimension. If it is compactified (a circle), the solution with the appropriate boundary conditions rather takes the form

$$\phi \sim Q\cosh\left[m\left(y - \frac{L}{2}\right)\right] \quad 0 \leq y \leq L. \quad (16)$$

The existence of such a brane is compatible with compactification.

Let us consider the issue of the fermionic zero modes on such branes. Consider the following coupling of a bulk fermion to  $\phi$

$$g(\partial_A \phi)\bar{\psi}\gamma_A\psi. \quad (17)$$

This creates a  $\theta(y)$  function type mass term, which changes the sign across the brane

$$gQ\theta(y)\bar{\psi}\gamma_5\psi. \quad (18)$$

Due to the index theorem there is a localized zero mode on the brane, which at  $|y| \ll L/2$  takes the form

$$\psi = f(x_\mu)\exp(-g|y|Q). \quad (19)$$

This zero mode will persist for the nonzero mass of the  $\phi$  as well.

Note that in the above discussion we could have used antisymmetric 4-form field  $A_{\alpha\ldots\beta}$  instead of the scalar  $\phi$ . (Such fields are present in many brane constructions (e.g. see [9]). To the best of our knowledge, however, this is the first attempt of using them for localizing the chiral fermionic zero modes). The whole discussion would go through, except that the fermion couplings now would be modified as follows. The coupling to the brane is

$$Q \int dx_\alpha \wedge dx_\beta \wedge dx_\gamma \wedge dx_\delta A_{\alpha\beta\gamma\delta}, \quad (20)$$

and the fermions now couple to the 5-form field strength

$$F_{\alpha\beta\gamma\delta\omega} = \partial_{[\alpha} A_{\beta\gamma\delta\omega]} \quad (21)$$

which changes the sign across the brane  $F \sim \theta(y)$ . The fermions coupled to  $F$

$$F_{\alpha\beta\gamma\delta\omega}\epsilon^{\alpha\beta\gamma\delta\omega}\bar{\psi}\psi \quad (22)$$

will develop a zero mode on the brane.



## 5 Prototype Model

Now we are in a position to write down a simple prototype five-dimensional Lagrangian giving rise to the desired structure. The important interaction terms are

$$L = |\Lambda - X^3|^2 + 2M^4 \cos \chi + (b \sin \chi - m^2) |H|^2 + |H|^4 + X^2 r_f \bar{f} f + X^{*2} r_{f_c} \bar{f}_c f_c + g_f H \bar{f} f_c \quad (23)$$

plus the standard kinetic and gauge terms. Here  $X$  is a gauge-singlet scalar that breaks  $Z_3$  symmetry and produces three walls on a compactified dimension.  $X$  changes the phase by  $2\pi/3$  through each of the walls and creates a single zero mode from each species of fermions  $f, f_c$ . These fermions have quantum numbers of one SM generation. Moreover,  $f$  stands for  $SU(2) \otimes U(1)$ -doublet, while  $f_c$  stands for  $SU(2) \otimes U(1)$ -singlet states, respectively. The Yukawa couplings with  $H$  and  $X$  are assumed to have the  $SU(2) \otimes U(1)$  doublet and singlet structures, respectively. In conventional notations the zero modes coming from  $f$  are the left-handed quark and lepton doublets  $Q, L$  and the ones coming from  $f_c$  are the left-handed *antisinglets*  $u_c, d_c, e_c$ . The precise form of the interaction terms between  $X$  and  $\chi$  is not very important, provided that the  $\chi$ -wall gets stabilized on top of one of the  $X$ -walls. This can be achieved, for instance, by adding [10]

$$(b' \sin \chi) |X|^2 \quad (24)$$

with a positive  $b'$ . The above Lagrangian reproduces all the desired features discussed above. It has three identical branes compatible with periodic boundary condition, with one SM generation localized per brane. Plus a separate brane that breaks the electroweak symmetry.

## 6 Hierarchical SUSY Breaking from Multiple Branes

In this section we will argue that the presence of the identical “family branes” may result to an exponentially weak supersymmetry breaking, even though each individual brane, in isolation, may be supersymmetry preserving (i.e. BPS saturated). This is the case if the multiple-brane state is a non-BPS configuration. The idea that the observable SUSY breaking may be due to the fact that we live on a non-BPS brane, was first put forward in [5]. Here we show that in the present circumstances this breaking may be exponentially weak.

Before formulating a very general sufficient condition for such weak supersymmetry breaking, let us illustrate the main point in a toy example. We are looking for a model that gives rise to a BPS brane, but in which the states with two (or more) such branes are non-BPS. The simplest model of this type is the one with the spontaneously broken  $R$  symmetry (the symmetry is  $Z_N$ ). In four-dimensions

the superpotential can be chosen as [11]

$$W = \Lambda X - \frac{cX^{N+1}}{N+1} \quad (25)$$

where  $X$  is a chiral superfield.

This theory has stable domain wall solutions across which the phase of  $X$  changes by  $2\pi/N$ . Clearly, having  $N$  such domain walls is compatible with periodicity of the transverse coordinate. Because the superpotential changes through the wall, this system admits a nontrivial central extension [5]; it can be shown that the elementary wall is BPS saturated. For  $N \rightarrow \infty$ , the corresponding solutions can be found explicitly [11]. However, the  $N$ -wall state on the cylinder is not BPS saturated, generally speaking.<sup>3</sup> Thus, the  $N$ -wall state, breaks all supersymmetries.

Let us assume that the transverse coordinate is compactified on a circle of radius  $R (= L/2\pi)$ . The equilibrium state in such a case corresponds to  $N$  branes around the circle at equal distances between the neighbors. What is the strength of the resulting supersymmetry breaking? It is exponentially suppressed by the inter-brane distance

$$\sim e^{-Rm} \quad (26)$$

where  $m$  is a mass of the  $X$  quanta, the scale that sets the width of the brane (we ignore factors of order  $N$ , which may be important, however, in the large  $N$  case). This weakness is not difficult to understand. The wall is a field configuration, that approaches the vacuum state *exponentially rapidly* in the transverse coordinate. Thus, unless there are massless fields that can “carry away” the message about its presence, all the influence of the brane is exponentially suppressed at large distances. So is the resulting supersymmetry breaking.

This gives us a very general sufficient condition for exponentially suppressed supersymmetry breaking:

- 1) the presence of the BPS brane, whose stability is *not* due to massless fields in the theory;
- 2) the  $N$ -brane states should not be BPS saturated.

Note that it is very important that the stability of the brane is due to the topological charge, which is *not* a source of any massless bulk field. If this stability were due to some other charge coupled to some massless bulk field (e.g. as in the  $D$  brane case), the corresponding field would serve as a messenger between the branes, and the resulting SUSY breaking would be power suppressed.

This is the crucial difference which differentiates our mechanism from the conventional schemes, in which the SUSY breaking gets transmitted between the branes by some bulk messenger interactions (e.g. see [15],[14]).

Once again, it is important to understand that massless fields in question are those coupled to the stabilizing charge, and not other massless fields. For instance,

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<sup>3</sup>Note, however, that the central charge can still be defined, it does not vanish for  $N$ -wall states [12]. Due to this reason, in particular, the junction of  $N$  domain walls (in the *noncompactified* space) can be BPS saturated [13, 12].

in any realistic theory, there is at least one massless field, the graviton, coupled to the energy-momentum tensor of the brane. This, however, will not serve as a messenger for the SUSY breaking, since individually all branes are SUSY preserving, and only their exponentially suppressed interaction brakes supersymmetry.

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